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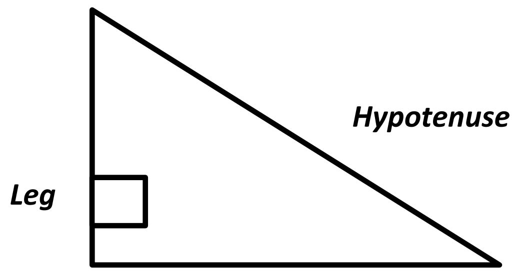
The Pythagorean Theorem Guide Notes

ANSWERS

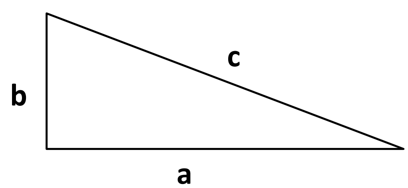
The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse of a right triangle.
In a right triangle, the side opposite the right angle is called ***the hypotenuse***. This side is always the longest side of a right triangle. The other two sides are called ***the legs*** of the triangle.



To find the length of any side of a right triangle when the lengths of the other two are known, you can use a formula by the Greek mathematician Pythagoras.

The Pythagorean Theorem

If **a** and **b** are the lengths of the legs of a right triangle, and **c** is the lengths of the hypotenuse, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$$c^2 = a^2 + b^2$$

Sample Problem 1: Find the length of the hypotenuse in the right triangle.

a. $a = 8$ $b = 15$ $c = ?$

$c^2 = a^2 + b^2$
$c^2 = 8^2 + 15^2$
$c^2 = 64 + 225$
$c^2 = 289$
$c = \sqrt{289}$
$c = 17$

Sample Problem 2: Find the length of the missing side of the right triangle.

a. $c = 10$ $a = 8$ $b = ?$

$c^2 = a^2 + b^2$
$b^2 = 10^2 - 8^2$
$b^2 = 100 - 64$
$b^2 = 36$
$b = \sqrt{36}$
$b = 6$

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If three positive integers (a, b, c) that represent the length of each side of a right triangle, satisfy the equation $\boldsymbol{c^2 = a^2 + b^2}$, it is called a ***Pythagorean triple.***

Sample Problem 3: Determine whether each set of numbers form a Pythagorean triple.

a. $(\mathbf{20, 21, 29})$ $\quad \boldsymbol{a = 20} \quad \boldsymbol{b = 21} \quad \boldsymbol{c = 29}$

$$\boldsymbol{c^2 = a^2 + b^2}$$
$$\mathbf{29^2 = 20^2 + 21^2}$$
$$\mathbf{841 = 400 + 441}$$
$$\mathbf{841 = 841}$$

Pythagorean triple

b. $(\mathbf{3, 6, 8})$ $\quad \boldsymbol{a = 3} \quad \boldsymbol{b = 6} \quad \boldsymbol{c = 8}$

$$\boldsymbol{c^2 = a^2 + b^2}$$
$$\mathbf{8^2 = 3^2 + 6^2}$$
$$\mathbf{64 = 9 + 36}$$
$$\mathbf{64 \neq 45}$$

Non Pythagorean triple

The statement than can be easily proved using a theorem is often called a corollary. The following corollary based on the Pythagorean Theorem can be used to determine whether a triangle is a right triangle.

If a and b are measures of the shorter sides of a triangle, then c is the measure of the longest side and $c^2 = a^2 + b^2$, then the triangle is a right triangle.

If $c^2 \neq a^2 + b^2$ then the triangle is not a right triangle.

if $c^2 < a^2 + b^2$ then the triangle is acute, and

if $c^2 > a^2 + b^2$ then the triangle is obtuse

Sample Problem 4: Determine whether the following side measures form right triangle.

a. $(\mathbf{4, 6, 9})$ $\quad \boldsymbol{a = 4} \quad \boldsymbol{b = 6} \quad \boldsymbol{c = 9}$

$$\boldsymbol{c^2 = a^2 + b^2}$$
$$\mathbf{9^2 = 4^2 + 6^2}$$
$$\mathbf{81 = 16 + 36}$$
$$\mathbf{81 \neq 52}$$

This is not a right triangle

b. $(\mathbf{16, 30, 34})$ $\quad \boldsymbol{a = 16} \quad \boldsymbol{b = 30} \quad \boldsymbol{c = 34}$

$$\boldsymbol{c^2 = a^2 + b^2}$$
$$\mathbf{34^2 = 16^2 + 30^2}$$
$$\mathbf{1156 = 256 + 900}$$
$$\mathbf{1156 = 1156}$$

This is a right triangle

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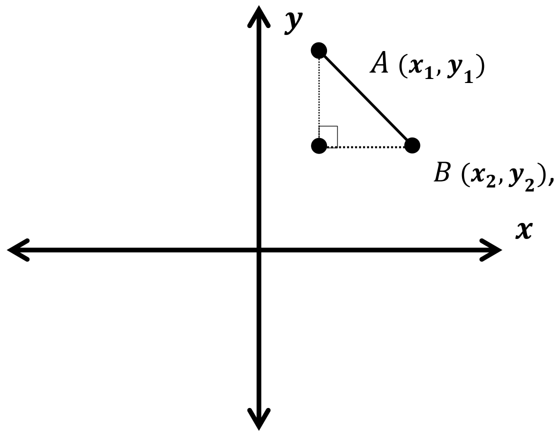
The Distance Formula

The distance formula is derived from the Pythagorean Theorem.
To find the distance between two points (x_1, y_1) and (x_2, y_2), all that you need to do is use the coordinates of these ordered pairs and apply the formula pictured below.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Sample Problem 5: Find the distance between the point at $(2, 3)$ and $(-4, 6)$.

(x_1, y_1) (x_2, y_2) $d = ?$
$(2, 3)$ $(-4, 6)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d =$
$\sqrt{((-4) - 2)^2 + (6 - 3)^2}$

$d = \sqrt{(-6)^2 + 3^2}$

$d = \sqrt{45}$

$d = 3\sqrt{5}$